**Code No: 132AB** **R16**

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**B. Tech I Year II Semester Examinations, February - 2025**
**MATHEMATICS-II**
**(Common to EEE, ECE, CSE, EIE, IT)**

**Time: 3 hours** **Max. Marks: 75**

**Note:** i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

## PART - A

**(25 Marks)**

1.a) Evaluate the Laplace transform of $f(t)=\begin{cases}4 & , \quad 0<t<1\\ 3 & , \quad t>1\end{cases}$. [2]

b) Define unit impulse function and find the Laplace of transform of unit impulse function. [3]

c) Evaluate $\int_0^\infty x^4 e^{-x^4} dx$. [2]

d) State and prove the symmetric property of Beta function. [3]

e) Define moments of inertia relative to $x$-axis and $y$-axis. [2]

f) Evaluate $\int_0^\infty\int_0^\infty e^{-(y+x)} dy\, dx.$. [3]

g) Verify whether $\bar{f}=(y+z)\bar{i}+(z+x)\bar{j}+(x+y)\bar{k}$ is irrotational vector or not? [2]

h) Determine $\nabla \bullet \left(r^3 \bar{r}\right)$, where $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$, $r=|\bar{r}|$. [3]

i) Define surface integral. [2]

j) Prove or disprove by Stokes theorem, $\int_s \left(\nabla\times\bar{F}\right)\bullet\bar{n}\, ds=0$, where $\bar{F}$ is any vector over $S: x^2+y^2+z^2=1$. [3]

## PART - B

**(50 Marks)**

2.a) Find the Laplace transform of the triangular wave function of period $2a$ given by
$$f(t)=\begin{cases} t & , \quad 0<t<a\\ 2a-t & , \quad a<t<2a\end{cases}$$

b) Use convolution theorem to determine $L^{-1}\left[\dfrac{1}{(s^2+1)(s^2+4)}\right]$. [4+6]

**OR**

3.a) What is the advantage of Laplace transform over other methods?

b) Use the method of Laplace transforms to solve $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}-3y=\sin x$ subject to the conditions $y(0)=0, \left(\dfrac{dy}{dx}\right)_{x=0}=0$. [2+8]

4.a) Express the integral $\int_0^{\infty} e^{-ax}x^{m-1}\sin bx\,dx$ in terms of Gamma function, where $m, a, b$ are constants.

b) Prove or disprove: $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta \times \int_0^{\pi/2}\frac{1}{\sqrt{\sin\theta}}\,d\theta = \pi$. [5+5]

**OR**

5.a) Define Beta function.

b) Prove that $\int_0^{\infty}\frac{1}{\left(e^{x}+e^{-x}\right)^{n}}dx = \frac{1}{4}\beta\left(\frac{n}{2},\frac{n}{2}\right)$ and hence evaluate $\int_0^{\infty}\operatorname{sech}^8 x\,dx$. [2+8]

6.a) Compute $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dx\,dy$ by transforming into polar coordinates.

b) Find the area bounded by the curves $y^2 = x^3$ and $x^2 = y^3$. [6+4]

**OR**

**7.** Determine the triple integral $\iiint_V \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ taken over the volume bounded by the planes $x=0,\ y=0,\ z=0$ and the plane $x+y+z=1$. [10]

8.a) Show that $\operatorname{div}\left(\operatorname{grad}\{r^m\}\right) = m(m+1)r^{m-2}$, where $m$ is a constant, $\bar{r} = x\bar{i}+y\bar{j}+z\bar{k},\ r=|\bar{r}|$.

b) Prove that the vector $\bar{f} = (x^2-yz)\bar{i}+(y^2-zx)\bar{j}+(z^2-xy)\bar{k}$ is irrotational vector. Hence find the scalar potential $\phi$ such that $\bar{f} = \nabla\phi$. [4+6]

**OR**

9.a) Compute the directional derivative of the function $\phi = x^2yz+4xz^2$ at $(1,-2,-1)$ in the direction of $2\bar{i}-\bar{j}-\bar{k}$.

b) Prove or disprove that $\operatorname{curl}\left(\frac{\bar{a}\times\bar{r}}{r^3}\right) = \frac{3\bar{r}(\bar{r}\bullet\bar{a})}{r^5} - \frac{\bar{a}}{r^3}$, where $\bar{a}$ is a constant vector. [4+6]

10.a) Find work done in moving a particle in the force field $\bar{F} = 3x^2\bar{i}+\bar{j}+z\bar{k}$ along the straight line from $(0,0,0)$ to $(2,1,3)$.

b) Evaluate $\int_S \left(\nabla\times\bar{f}\right)\bullet\bar{n}\,ds$, where $\bar{F} = (x+2y)\bar{i}-3z\,\bar{j}+x\,\bar{k}$ and $S$ is the surface of the plane $2x+y+2z=6$ bounded by the coordinate planes $x=0,\ y=0$ and $z=0$. [5+5]

**OR**

11.a) State Stokes theorem.

b) Verify the Stoke's theorem for the vector field $\vec{F} = x\,\bar{i}+z^2\bar{j}+y^2\bar{k}$ over the plane surface $x+y+z=1$ lying in the first octant. [2+8]

**--ooOoo--**